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A COMPACT HARDWARE REALIZATION FOR APPROXIMATE DIVISION.(U)
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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
LINCOLN LABORATORY

A COMPACT HARDWARE REALIZATION FOR APPROXIMATE DIVISION

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Group 27

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LEXINGTON MASSACHUSETTS

ABSTRACT

The design and implementation of hardware for approximate digital division is described. Using standard, commercially-available T^2L components, a compact divider has been developed which is well-suited for use within 5 MHz systems having three-state busses. After forming double-length denominator reciprocals by table look-up to high accuracies, single and extended-precision quotients are made available by consecutive numerator multiplications using a single-chip array multiplier.

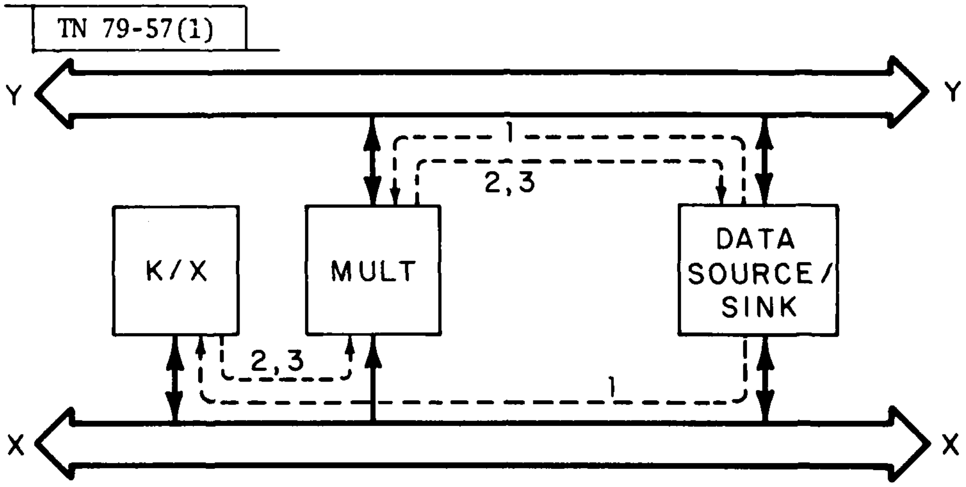
CONTENTS

I. INTRODUCTION

For a large number of digital signal processing algorithms, it has generally been true that the infrequent need for a full-precision 2's complement divide capability has re-resulted either in this function being implemented in software, or else denominator reciprocal values being prestored in memory for later add-shift numerator multiplication to achieve "real time" division. For those applications wherein a fast, compact hardware divide capability is essential, the choices have been somewhat limited. For example, one may implement either a successive-approximation or a non-restoring division algorithm using one or two handfuls of MSI T^2L devices or utilize a single-chip multiplier/divider (e.g., the MMI #67516), but in either case the expected quotient calculation time exceeds two microseconds.

A table look-up method is described in this paper for forming the approximate reciprocal of any 16-bit 2's complement input using 12 MSI T^2L chips (plus SSI gates). Access time is slightly more than 100 ns. When used together with a single-chip 16 x 16 array multiplier, single-precision quotient calculations require about 250 ns.

Figure 1 illustrates how the scaled reciprocal function (K/X) may be used within a two-bus μC structure ($K = 2^{-15}$).



INST No.	ACTION
1	K/X ← X, MULT ← Y
2	MULT ← K/X, Y ← MULT
3	MULT ← K/X, Y ← MULT

Fig. 1. Two-bus microcomputer structure showing data flow for single and double-precision 'pseudodivides'.

In the context of a 5 MIPs microcomputer currently being developed, nominal 200 ns instruction times provide generous overhead allowances for data bus transfers and control delays, while expanding the total alloted time for reciprocal estimation and multiplication to 400 ns. All functional blocks shown in Figure 1 communicate via three-state data buses X and Y, where K/X is equipped with an input register, the multiplier is fully registered on both input and output ports, and the data source/ sink is intended to represent one of several possible two-ported data elements, such as a data memory or CPE register file. After operands X and Y are transferred and K/X is formed during instruction 1, a single-precision quotient is generated during instruction 2 by loading K/X into the multiplier, doing the multiplication, and delivering the result to the data sink. Instruction 3 accesses the extended-precision portion of the K/X table for multiplication by the same numerator Y to complete the formation of an approximate double-precision quotient.

When the reciprocal table is used for the "pseudodivide" operations described here, scaling factor K is eliminated by exploiting the output select feature (MSP/LSP) of the multiplier to rescale the resultant output, $\hat{Q} = Y/X$. A transfer function such as K/X whose hardware is internally registered and equipped with a three-state port may be applied within single-bus structures as well as dedicated (non-programmable) hardware.

II. K/X APPROXIMATION

There is ample incentive to minimize the required amount of table storage space for a reciprocal function which will accommodate 16-bit signed 2's complement inputs. Even a single-precision (16-bit) unsampled table requires 2^{20} bits of storage or sixty-four 16K-bit programmable read-only memory devices, apart from support circuitry. By imposing a reasonable error constraint, we can piecewise-approximate the K/X curve and get a dramatic table memory (PROM) saving. Thus, assuming for now that output data precision is not an issue (i.e., assuming "infinite precision"), we seek to approximate K/X with closely spaced points in the region of steepest slope (smallest X), but to use progressively more sparse spacing of points as the slope diminishes and $|X|$ approaches one. For the moment, let us consider only the positive region of the function $R = K/X$. Figure 2 shows the alternate point sampling that would be likely within a region of steeper slope.

Reduction of the required number of table entries involves 1) the choice of compromise K/X values, and 2) the choice of progressively wider K/X sample spacing as X increases from decimal zero to one. A PROM table addressing algorithm is needed which will compress an unsampled, 2^{16}-word table into a much smaller one. For logical simplicity, let the address quantization be binary-weighted.

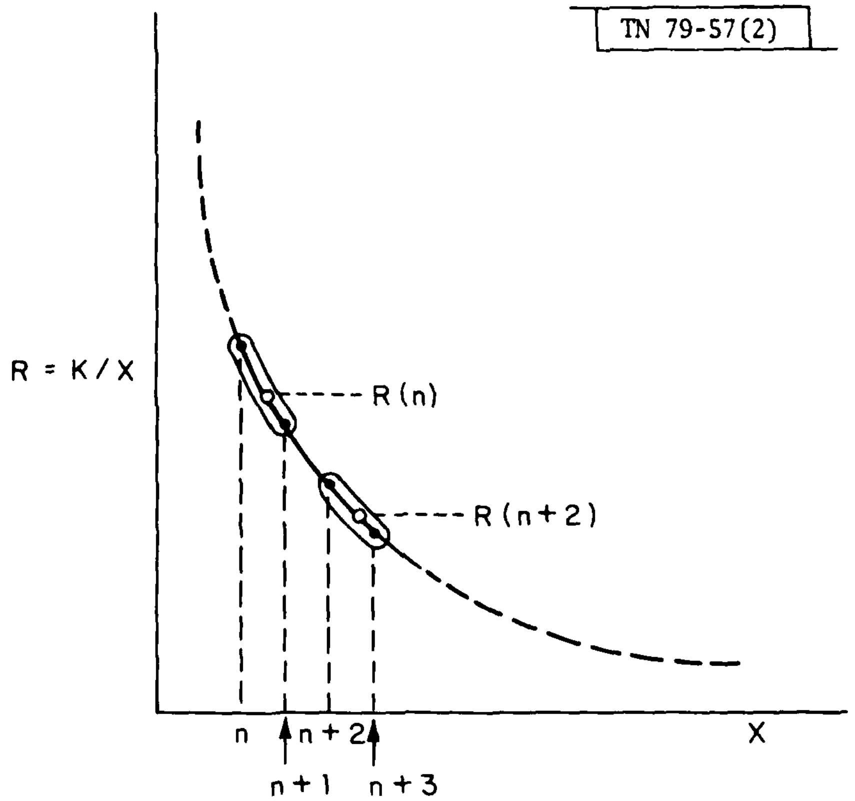


Fig. 2. Illustration of alternate point sampling for R = K/X ($K = 2^{-15}$, $X = n \cdot 2^{-15}$).

For the positive region, assuming 2's complement fractional inputs, define index $n = 0 \rightarrow 32767$, $X = n \cdot 2^{-15}$, and $K = 2^{-15}$ such that

$$R = K/X = 2^{-15}/n \cdot 2^{-15} = 1/n \Bigg|_{n=0}^{32767} \qquad (1)$$

If the quantization factor (Δ) represents the number of contiguous X points spanned for a given table address, E_A denotes normalized approximation error, and $\hat{R}$ is the approximate value for R over the same span, then for uniform normalized peak error,

$$E_A = (1/n-\hat{R})/(1/n) = \left[\hat{R}-1/(n+\Delta-1)\right]/\left[1/(n+\Delta-1\right] \qquad (2)$$

from which

$$\hat{R} = 2/\left[2n+(\Delta-1)\right] \qquad (3)$$

and

$$E_A = (\Delta-1)/\left[2n+(\Delta-1)\right] \qquad (4)$$

Reciprocal function K/X can be organized into several neighboring zones, each having its own unique Δ. As $X = n \cdot 2^{-15}$ increases from $n = 1$, this first zone (#1) requires that $\Delta = 1$ (from (4), $\Delta = 2$ would imply that $E_A = 0.33$ at $n = 1$, an unacceptably large error). Zone 1 is somewhat special in that $\Delta = 1$ results in a 1:1 relationship between table addresses and data, and no approximation error exists. The end point for zone 1 may be deduced by calculating the beginning value of n for zone 2, subject to an error constraint. From (4), for

$$E_A \leq (\Delta-1)/\left[2n+(\Delta-1)\right], \quad n \geq (1-E_A)(\Delta-1)/2E_A \qquad (5)$$

Thus if we require $E_A \leq 1/2\%$, then $n \geq 100$. Using the nearest larger binary value, choose n = 128 as the starting point of zone 2.

To minimize the number of table entries as $X = n \cdot 2^{-15}$ increases toward 1, we wish to increase Δ to 2Δ at the earliest opportunity. For uniform peak error, the zone boundary can be determined by solving (4) for n when $\Delta = 2\Delta$, where n_i and n_{i+1} are the first address points in zones i and i+1, respectively:

$$E_A = (\Delta_i-1)/[2n_i+(\Delta_i-1)] = (2\Delta_i-1)/[2n_{i+1} + (2\Delta_i-1)] \qquad (6)$$

$$(n_{i+1})/n_i = (2\Delta_i-1)/(\Delta_i-1) \qquad (7)$$

The starting point of zone 3 ($\Delta = 4$) determines the end point of zone 2, and from (7) is $n_{i=3} = 128\ (4-1)/(2-1) = 384$. Equation 7 determines the beginning points of subsequent zones in a similar manner, with uniform peak error throughout, predetermined by the starting point of zone 2. Because zone 2 begins at n = 128, universal $E_{A_{max}} = 0.00389$. Table 1 shows the range of n values for the 9 zones needed to generate the positive portion of an approximate K/X function. Fortunately, zone 9 may be merged with zone 8 by letting the last stored $\hat{R}$

TABLE 1

NUMBER RANGE AND QUANTIZATION FACTORS (Δ)

FOR A 9-ZONE APPROXIMATE RECIPROCAL FUNCTION

				Input X*														
n	Δ	Zone	S	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
0 → 127	1	1	0 ⋮ 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 1	0 1	0 1	0 1	0 1	0 1	0 ⋮ 1
128 → 383	2	2	0 ⋮ 0	0 0	0 0	0 0	0 0	0 0	0 0	0 1	1 0	0 1	0 1	0 1	0 1	0 1	0 1	0 ⋮ 1
384 → 895	4	3	0 ⋮ 0	0 0	0 0	0 0	0 0	0 0	0 1	1 1	1 0	0 1	0 1	0 1	0 1	0 1	0 1	0 ⋮ 1
896 → 1919	8	4	0 ⋮ 0	0 0	0 0	0 0	0 0	0 1	1 1	1 1	1 0	0 1	0 1	0 1	0 1	0 1	0 1	0 ⋮ 1
1920 → 3967	16	5	0 ⋮ 0	0 0	0 0	0 0	0 1	1 1	1 1	1 1	1 0	0 1	0 1	0 1	0 1	0 1	0 1	0 ⋮ 1
3968 → 8063	32	6	0 ⋮ 0	0 0	0 0	0 1	1 1	1 1	1 1	1 1	1 0	0 1	0 1	0 1	0 1	0 1	0 1	0 ⋮ 1
8064 → 16255	64	7	0 ⋮ 0	0 0	0 1	1 1	1 1	1 1	1 1	1 1	1 0	0 1	0 1	0 1	0 1	0 1	0 1	0 ⋮ 1
16256 → 32639	128	8	0 ⋮ 0	0 1	1 1	1 1	1 1	1 1	1 1	1 1	1 0	0 1	0 1	0 1	0 1	0 1	0 1	0 ⋮ 1
32640 → 32767	256	9	0 ⋮ 0	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	0 1	0 1	0 1	0 1	0 1	0 1	0 ⋮ 1

*2's complement, fractional

value span n = 32512 → 32767, Δ = 256. Each zone requires 128 storage locations, and the 8-zone arrangement reduces the needed storage by a factor of 32 with respect to unsampled storage. For both positive and negative regions of the function K/X, with approximation error of less than 1/2% and a data wordlength of 16 bits, 32K bits of PROM table storage is sufficient.

Figure 3 shows the function R(x) (at somewhat distorted scale) for which the approximate function $\hat{R}(x)$ is to be computed and stored in PROM tables. The intent here is to illustrate the numerical range of interest, and to take note of departures from the ideal curve attributable to 2's complement asymmetry along with a limit when X = 0. For the positive region, X ranges from 0 to $1-2^{-15}$, with R(0) and $R(2^{-15})$ constrained to the largest representable positive fraction, $1-2^{-15}$. For the negative region, X ranges from -2^{-15} to -1, and R(x) is the straightforward scaled reciprocal. Excluding R(0) and $R(2^{-15})$, the asymmetry about the origin may be expressed as

$$R(A^{+}) = -R(A^{+}-1), \qquad (8)$$

where A^{+} is any given address within the positive region.

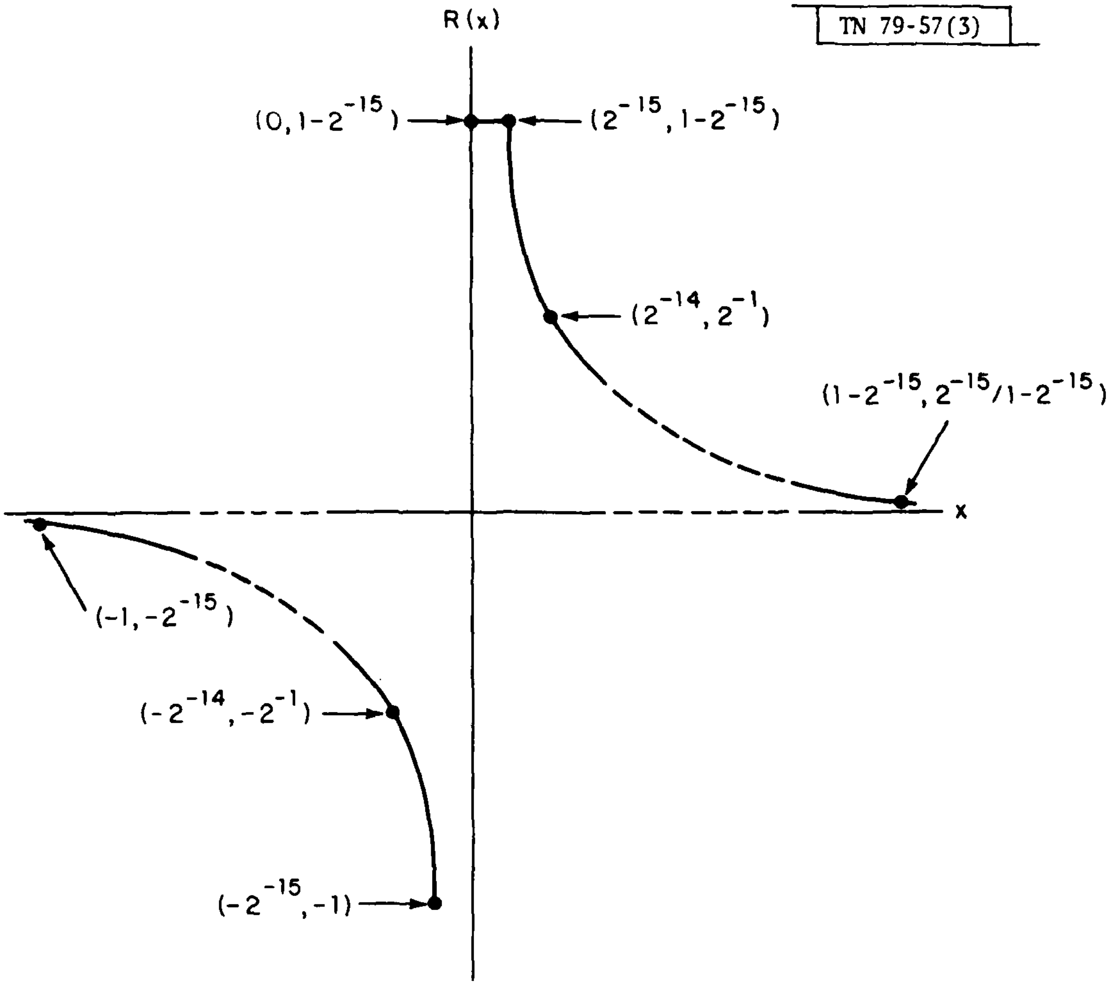


Fig. 3. Numerical range of interest for reciprocal function R(x).

III. K/X QUANTIZATION

It is important to consider the accuracy limits of $R = K/X$, i.e., the precision with which one may represent the scaled reciprocal. Let us define the normalized quantization error as

$$E_Q = (R-R_Q)/R, \qquad (9)$$

where R is the exact (infinite precision) value of K/X and R_Q is its quantized value. As indicated in Fig. 4, the absolute amount of quantization error in a single-precision R table may approach the value of the LSB (2^{-15}). But on a percentage basis, this error has the least impact when the absolute value of X is small, i.e., when R is large.

Tables 2 and 3 indicate the expected worst-case total errors (approximation plus quantization) for single and double-precision realizations, respectively. Recalling from (1) that

$$R = K/X = 1/I \Big|_{I=0}^{I_{max}} \qquad (10)$$

where

$$X = I \cdot 2^{-15} \quad , \qquad (11)$$

both tables show normalized error as a function of I_{max} and X_{max}, together with the input values (I_{pk} and X_{pk}) at which total error E_T is maximum. As expected, Table 2 shows approxi-

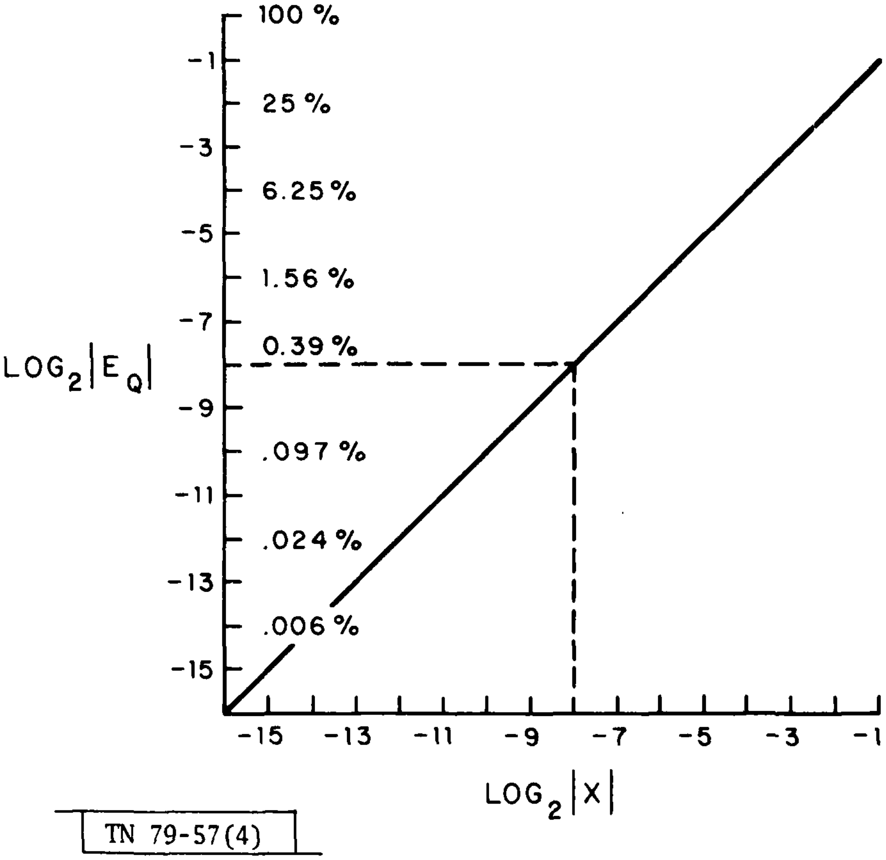


Fig. 4. Maximum quantization error (E_Q) versus denominator (x) magnitude for single-precision reciprocal, R.

TABLE 2

ERROR ANALYSIS FOR SINGLE-PRECISION K/Y

IMAX	IPK	XMAX	XPK	EA	EQ	ET
1	1	0.0	0.000030518	0.0	0.0	0.0
2	1	0.000030518	0.000030518	0.0	0.0	0.0
4	4	0.000091553	0.000122070	0.0	0.000061	0.000061
8	6	0.000213623	0.000183105	0.0	0.000091	0.000091
16	12	0.000457764	0.000366211	0.0	0.000305	0.000305
32	30	0.000946045	0.000915527	0.0	0.000824	0.000824
64	59	0.001922607	0.001800537	0.0	0.001709	0.001709
128	114	0.003875732	0.003479004	0.0	0.003387	0.003387
256	247	0.007781982	0.007537842	0.002028	0.007005	0.009033
512	489	0.015594482	0.014923096	0.003064	0.014025	0.017090
1024	993	0.031219482	0.030303955	0.003516	0.027734	0.031250
2048	1937	0.062469482	0.059112549	0.003859	0.050829	0.054687
4096	3649	0.124969482	0.111358643	0.002051	0.107323	0.109375
8192	6561	0.249969482	0.200225830	0.002357	0.196862	0.199219
16384	10945	0.499969482	0.334014893	0.002870	0.329161	0.332031
32768	16385	0.999969482	0.500030518	0.003861	0.496139	0.500000

TABLE 3

ERROR ANALYSIS FOR DOUBLE-PRECISION K/X

IMAX	IPK	XMAX	XPK	EA	EQ	ET
1	1	0.0	0.000030518	0.0	0.0	0.0
2	1	0.000030518	0.000030518	0.0	0.0	0.0
4	1	0.000091553	0.000030518	0.0	0.0	0.0
8	1	0.000213623	0.000030518	0.0	0.0	0.0
16	1	0.000457764	0.000030518	0.0	0.0	0.0
32	1	0.000946045	0.000030518	0.0	0.0	0.0
64	1	0.001922607	0.000030518	0.0	0.0	0.0
128	1	0.003875732	0.000030518	0.0	0.0	0.0
256	129	0.007781982	0.003936768	0.003891	0.000000	0.003891
512	129	0.015594482	0.003936768	0.003891	0.000000	0.003891
1024	897	0.031219482	0.027374268	0.003891	0.000000	0.003891
2048	897	0.062469482	0.027374268	0.003891	0.000000	0.003891
4096	3969	0.124969482	0.121124268	0.003891	0.000001	0.003892
8192	8065	0.249969482	0.246124268	0.003891	0.000003	0.003894
16384	8065	0.499969482	0.246124268	0.003891	0.000003	0.003894
32768	32513	0.999969482	0.992218018	0.003906	0.000000	0.003906

mation error (E_A) to be zero for zone 1, and constraining X to be no larger than 2^{-8} results in quantization error (E_Q) of less than 0.4% as indicated in Fig. 4. Note also that while approximation error is minimal throughout the usable input range, quantization error rapidly becomes unacceptable with increasing X. However, for those applications requiring good accuracy over the full dynamic range of X, Table 3 shows that extended precision approximation of K/X yields improved results. Simply stated, a single-precision approximation to K/X can resolve to within 2^{-15} (the LSB), whereas a double-precision approach is accurate to with 2^{-30}. The penalty for extended precision is an additional multiply operation when doing a "pseudodivide".

The Fortran programs for generating Tables 2 and 3 are shown in Appendix A.

For single-precision results to have tolerable limits on error, some constraints must be imposed on the range of allowable divider inputs shown in Fig. 4. If, after compensating for scaling we say that 16-bit quotient

$$Q = Y/X \quad , \tag{12}$$

and avoidance of overflow requires that

$$|Y| < |X| \quad , \tag{13}$$

then for quantization error $E_Q < 0.4\%$ as chosen for this particular design,

$$|X| \leq 2^{-8} \tag{14}$$

and

$$|Q| \geq 2^{-7} \tag{15}$$

In the context of a given μC program requiring a fast divide, a priori knowledge and maintenance of the relative magnitudes of X and Y can permit straightforward, fast generation of single-precision quotients. If, however, a number of instructions are expended in first shifting divider operands, the hardware divide feature may have less appeal. For this reason, the K/X address map function has been designed to accommodate the full denominator (X) dynamic range, and permits the addressing of a double-precision (32-bit) K/X table simply by including two additional PROMs. The divider hardware then provides a single-precision $\hat{Q}$ in two instructions (400 ns), and an extended precision $\hat{Q}$ with one additional 200 ns instruction, subject only to the initial requirement for $|Y| < |X|$. If, on the other hand, we had elected to limit the input range to $|X| \leq 2^{-8}$ (see Fig. 4), a simple 256 x 16 PROM table with no address mapping would have sufficed.

IV. ADDRESS MAPPING

Figure 5 is a simplified block diagram showing the generation of K/X by cascading address map and table memory sections. The principal design task here is the derivation of a mapping function whose implementation will have reasonable speed and a minimal number of integrated circuits. By examining the 2's complement representation of X as described in Table 1, the logical relationships for address mapping may be deduced and minimized. Although only numbers for the positive region are considered here, the same mapping logic may be used for the negative region after inverting input word X under control of sign bit X_s. Figure 5 shows an 11-bit table address, where bit A_{10} determines the table region (+,-), address bits A_9 through A_7 control zone selection, and bits A_6 through A_0 constitute the zone address.

Let seven intermediate variables be defined as follows:

$$
\begin{aligned}
C_1 &= \overline{X}_8 X_7 + X_8 \overline{X}_7 \\
C_2 &= \overline{X}_9 X_8 X_7 + X_9(\overline{X}_8 + \overline{X}_7) \\
C_3 &= \overline{X}_{10} X_9 X_8 X_7 + X_{10}(\overline{X}_9 + \overline{X}_8 + \overline{X}_7) \\
C_4 &= \overline{X}_{11} X_{10} X_9 X_8 X_7 + X_{11}(\overline{X}_{10} + \overline{X}_9 + \overline{X}_8 + \overline{X}_7) \\
C_5 &= \overline{X}_{12} X_{11} X_{10} X_9 X_8 X_7 + X_{12}(\overline{X}_{11} + \overline{X}_{10} + \overline{X}_9 + \overline{X}_8 + \overline{X}_7) \\
C_6 &= \overline{X}_{13} X_{12} X_{11} X_{10} X_9 X_8 X_7 + X_{13}(\overline{X}_{12} + \overline{X}_{11} + \overline{X}_{10} + \\
&\quad \overline{X}_9 + \overline{X}_8 + \overline{X}_7) \\
C_7 &= X_{14} X_{13} X_{12} X_{11} X_{10} X_9 X_8 X_7 \qquad (16)
\end{aligned}
$$

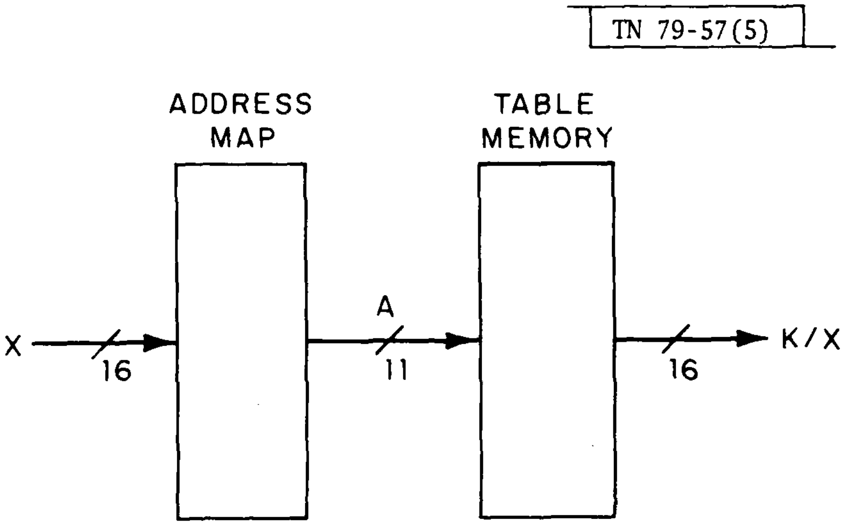


Fig. 5. Simplified reciprocal function generator block diagram.

Then

$$A_{10} = X_s \qquad , \tag{17}$$

and zone select bits are

$$\begin{aligned}
A_9 &= \overline{X}_{14}\overline{X}_{13}\overline{X}_{12}C_4 + \overline{X}_{14}\overline{X}_{13}C_5 + \overline{X}_{14}C_6 \\
&\quad + \overline{X}_{14}X_{13}X_{12}X_{11}X_{10}X_9X_8X_7 + X_{14} \\
A_8 &= \overline{X}_{14}\overline{X}_{13}\overline{X}_{12}\overline{X}_{11}\overline{X}_{10}C_2 + \overline{X}_{14}\overline{X}_{13}\overline{X}_{12}\overline{X}_{11}C_3 \\
&\quad + \overline{X}_{14}C_6 + \overline{X}_{14}X_{13}X_{12}X_{11}X_{10}X_9X_8X_7 + X_{14} \\
A_7 &= \overline{X}_{14}\overline{X}_{13}\overline{X}_{12}\overline{X}_{11}\overline{X}_{10}\overline{X}_9C_1 + \overline{X}_{14}\overline{X}_{13}\overline{X}_{12}\overline{X}_{11}C_3 \\
&\quad + \overline{X}_{14}\overline{X}_{13}C_5 + \overline{X}_{14}X_{13}X_{12}X_{11}X_{10}X_9X_8X_7 + X_{14}
\end{aligned} \tag{18}$$

The required zone address bits are shown in Table 4 below.

TABLE 4
ZONE ADDRESS BITS AS A FUNCTION OF ZONE
NUMBER AND INPUT VARIABLES X,C

Zone Zone bits	1	2	3	4	5	6	7	8,9
A_6	X_6	$\overline{X}_7$	C_1	C_2	C_3	C_4	C_5	$(C_6 + C_7)$
A_5	X_5	X_6	$\overline{X}_7$	C_1	C_2	C_3	C_4	$(C_5 + C_7)$
A_4	X_4	X_5	X_6	$\overline{X}_7$	C_1	C_2	C_3	$(C_4 + C_7)$
A_3	X_3	X_4	X_5	X_6	$\overline{X}_7$	C_1	C_2	$(C_3 + C_7)$
A_2	X_2	X_3	X_4	X_5	X_6	$\overline{X}_7$	C_1	$(C_2 + C_7)$
A_1	X_1	X_2	X_3	X_4	X_5	X_6	$\overline{X}_7$	$(C_1 + C_7)$
A_0	X_0	X_1	X_2	X_3	X_4	X_5	X_6	$(\overline{X}_7 + C_7)$

The foregoing expressions are the result of examining the relationships between binary input values (X) and required address values (A), with particular emphasis upon the X states which mark the zone boundaries. Simply stated, knowledge of the zone within which any input X falls determines 1) the page in table memory containing the appropriate K/X value, and 2) the modulus by which changes in X advance the address count. Consider, for example, the bottom row of table 4. As we proceed along the K/X curve from zone 1 to zone 8, the table address (with LSB = A_0) is incremented modulo 1,2,4,8,... etc., as expected.

V. IMPLEMENTATION

The schematic diagram of Fig. 6 is the result of considering several approaches to minimization of the address mapping logic, using standard components. While intuition initially suggested the use of barrel shifters or FPLAs, the use of fast PROMs in conjunction with programmable multiplexers yielded the most compact design. The zone address pattern of Table 4 is suggestive of a large multiplexer, and indeed the PMUXs of Fig. 6 permit generation of all zone address bits except A_6. PROMs A and B provide all the variables of Eqs. 16 and 18, plus A_6. The 16K-bit table storage PROMs represent the best density/speed combination currently available, and provide needed three-state outputs. The 25LS2519 input registers are particularly well suited to this application because of their output invert feature. While the address mapping logic is configured to accept binary inputs in magnitude format, its usefulness is extended by the input register's ability to supply the 1's complement of negative inputs under sign bit control. This same sign bit (X_{15}) selects the table region (+,-) via address bit A_{10}.

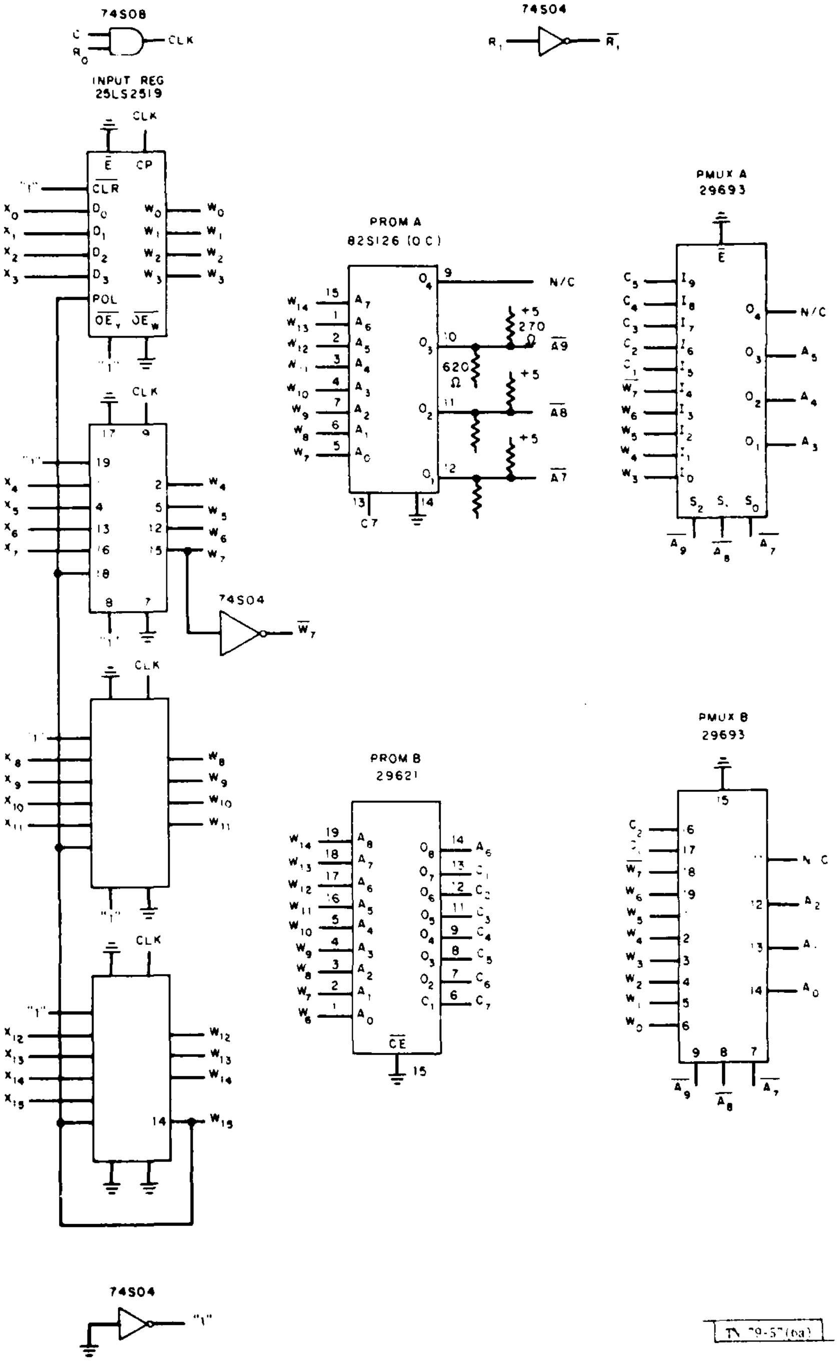


Fig. 6. Logic diagram for double-precision approximate reciprocal function.

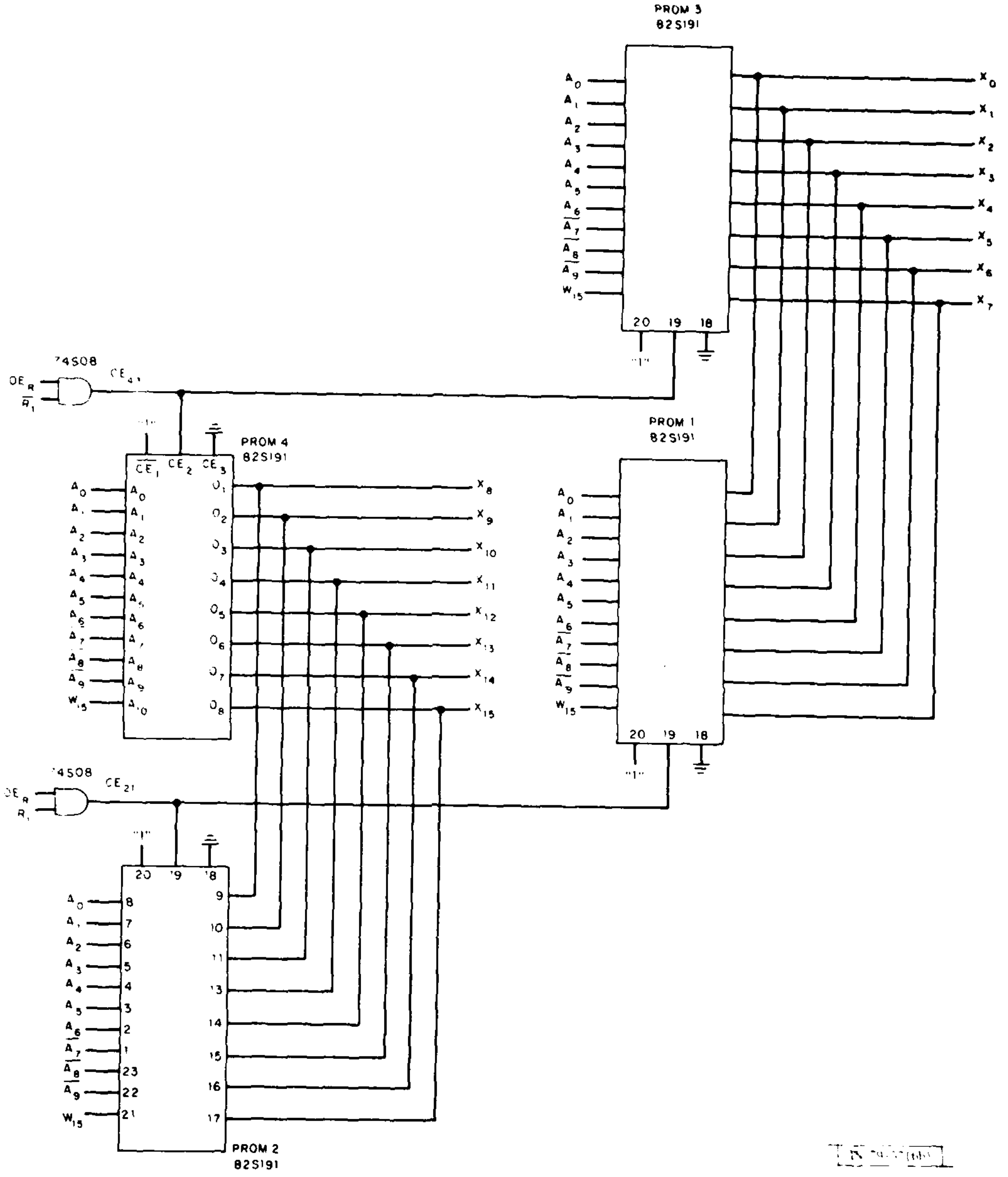


Fig. 6. Continued.

One additional hardware trick is needed to realize the zone 8,9 column of Table 4. As mentioned earlier, a residual zone 9 ($32640 \leq n \leq 32767$) exists for which intermediate variable C_7 is uniquely true. This small region can be merged with zone 8 with no discernible error penalty. In Table 4, the presence of the C_7 terms within the zone 8,9 column is the assertion of zone 9. However, because PMUX input ports are already fully utilized (so much so that A_6 is generated via PROM B instead of PMUX A or B), another method for including the contribution of C_7 must be used. This is done by storing zone select bits ($A_9 \rightarrow A_7$) within PROM A in their complemented form, and applying C_7 to PROM A as a chip enable. The programming of PMUXs and table PROMs is then done in accordance with $\bar{A}_9,\bar{A}_8,\bar{A}_7$ instead of A_9,A_8,A_7, and whenever C_7 is asserted, zone selection is constrained to zone 8 via the PROM A chip enable. Zone address bits A_0 through A_6 are unaffected.

A. PROM and PMUX Programming

Referring to Fig. 6, PROM A is required for zone selection within the table PROMs, whereas PROM B is used principally for the development of intermediate address mapping variables. With the exception of bit A_6, PMUXs A and B generate the table address shown in Table 4. The requisite data patterns to be stored were written in accordance with Tables 1 and 4 and Eqs. 16 and 18, and are shown in Appendix B. For purposes of electrical programming, the 29693 PMUX is viewed as a 256 x 4 PROM.

From Fig. 6, Table PROMs 4 and 3 are selected for single-precision readouts, while PROMS 2 and 1 contain extended-precision data. A Fortran program called TABLE1 (Appendix C) calculates the decimal, integer, and hex values required for the single-precision PROM pair, based on parameters such as number of zones and the quantization factor within each zone. TABLE1 outputs four disk files entitled BYTE43 DEC, BYTE43 INT, BYTE4 HEX, and BYTE3 HEX. The first two outputs provide fractional and integer decimal representations of the entire single-precision list for checking purposes. Because these files are somewhat voluminous, they are not included in the appendix. The remaining two files are the result of splitting the integer file into two pieces and adopting the hexadecimal format, such that the resultant tables are directly useful as data files for programming the two byte-wide memories, PROMs 3 and 4. These listings are shown in Appendix D.

Fortran program TABLE2 (Appendix E) is nearly the same as TABLE1, except that its purpose is to generate the data patterns needed for extended-precision PROMs 2 and 1. It does so by calculating to high precision all the table values needed for double-precision, then deducting the values already stored in PROMs 4 and 3, and using the residue as data for PROMs 2 and 1. The TABLE2 output files are labeled BYTE21 DEC, BYTE21 INT, BYTE2 HEX, and BYTE1 HEX. The latter two are shown in Appendix F. As a point of interest, program TABLE2 resolves each table entry to within 2^{-30} instead of 2^{-31}, because a sign bit must be included within both the most and least significant 16-bit words made available on the K/X data bus.

The array multiplier to be used in conjunction with the reciprocal function unit for doing 'pseudodivides' is the TRW model MPY-16HJ. Figure 7 shows the block diagram for this single-chip LSI device which can typically provide 32-bit products within 125 ns. Two particularly useful features in the present context are fully-registered inputs and outputs, plus the availability of three-state outputs. Further reference to these features will be made when defining multiplier operating modes.

B. Reciprocal Function Modes

Control of the reciprocal function unit involves some simple choices relating to 1) the status of the three-state port (input vs output), 2) whether or not the data register stores new information, and 3) single or extended-precision output (MSR or LSR). Table 5 below outlines some basic modes based upon the availability of an input clock and control bits R_0, R_1, and OE_R.

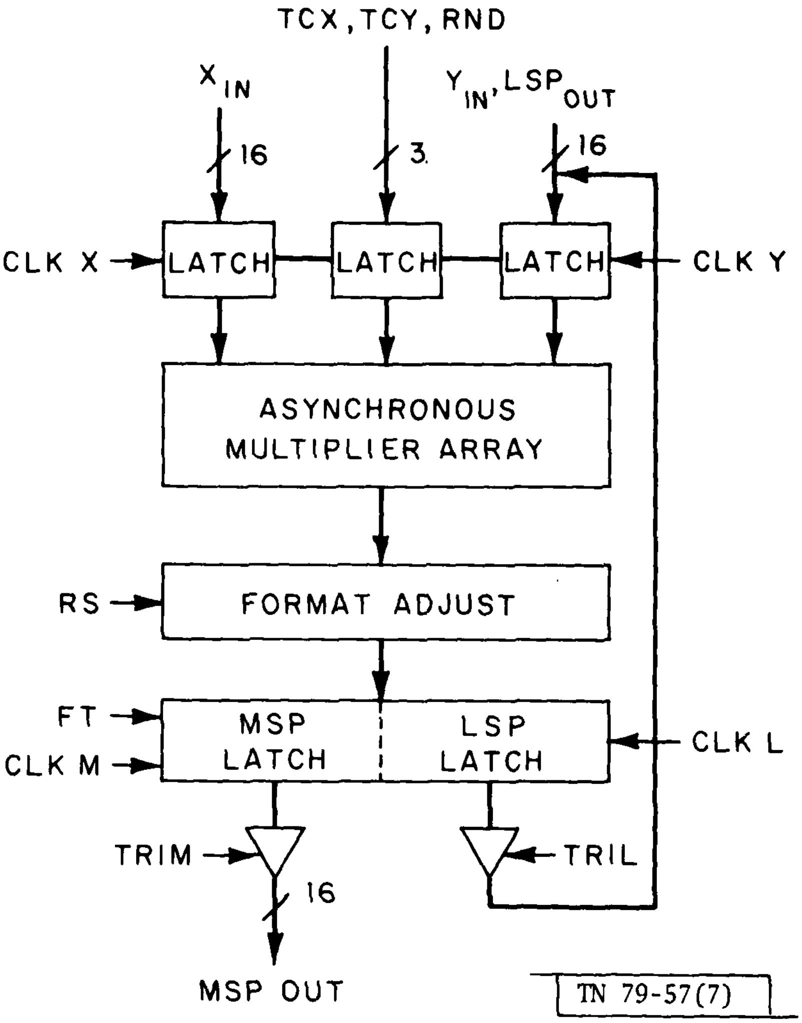


Fig. 7. TRW MPY-16 HJ 16 x 16 bit parallel multiplier block diagram.

TABLE 5
ESSENTIAL CONTROL STATES FOR RECIPROCAL FUNCTION UNIT

OE_R	R_1	R_0	CLK	CE_{43}	CE_{21}	Mode
1	0	0	0	1	0	Read MSR
1	1	0	0	0	1	Read LSR
0	X*	0	0	0	0	Hold
0	X	1	↑	0	0	Calc/Hold

*X = don't care

As shown in Fig. 6, R_0 gates an external clock (C) and controls updating of the data register. R_1 determines the MSR/LSR choice when accessing the table PROMs in conjunction with bus control OE_R. Referring to the three instructions of Fig. 1 for forming approximate quotients, the corresponding reciprocal unit commands in accordance with Table 5 are Calc/Hold, Read MSR, Read LSR.

C. Multiplier Modes

Given a set of externally-applied clocks and control states, the internal registers and three-state capability of the MPY-16HJ may be used to full advantage for calculating approximate quotients. Table 6 defines several useful multiplier modes. The use of separate input and output clocks (C_i and C_o) is assumed, three-state ports become outputs when TRIM or TRIL are zero, and X denotes "don't care" conditions.

TABLE 6
CONTROL STATES FOR TRW MULTIPLIER

CLK_x	CLK_y	$CLK_{l,m}$	TRIM	TRIL	Mode
C_i	0	C_o	1	1	Load X
0	C_i	C_o	1	1	Load Y
C_i	C_i	C_o	1	1	Load X,Y
X	X	0	1	1	Hold
X	X	0	0	1	Read MSP
X	X	0	1	0	Read LSP

Multiplication occurs on any Load instruction, and the resultant product is retained in the output register(s). The Hold condition retains data in the output registers, and is generally used when no multiplier outputs are driving the bus. The Read operations simply amount to suppressing the output register clocks and enabling the appropriate three-state driver to access either the most or least significant product.

Table 6 depicts only a subset of the total number of unique operations possible using the MPY-16HJ multiplier. An underlying assumption of Fig. 1 is that some of the modes of Table 6 may be concatenated within any given 200 ns nominal microcomputer instruction cycle. In particular, for the three-instruction loop of Fig. 1, the multiplier does Load Y, Load X/Read LSP, and Load X/Read LSP operations while the reciprocal unit is doing Calc/Hold, Read MSR, and Read LSR.

The split multiplier instruction Load X/Read LSP is faster than others such as Load Y/Read LSP because the Y bus is not shared between input and output traffic, and no three-state output enable delay is incurred. Another multiplier feature which may be exploited is the feedthrough provision. If local storage of a product (in the multiplier output register) is not needed, but instead a result is to be stored at some remote point, the FT line on the MPY-16HJ renders the output register transparent, making the multiplier output available earlier.

When doing division, the scale factor (K) associated with the single-precision reciprocal function is effectively removed by shifting the multiplier output by 2^{+15}, i.e., by interpreting the LSP in accordance with MSP binary weights. For extended precision calculations, the reciprocal table output is interpreted as being unscaled.

VI. RESULTS

Using commercial grade integrated circuits, bench tests have shown the clock-to-output throughput delay of the one reciprocal function built to be no greater than 110 nanoseconds at 25°C. The expected delay value over the commercial temperature range should not exceed 165 ns. For those instances where X has been stored in the K/X holding register for a long time (e.g., when fetching an extended-precision result), access via table PROM chip enable may take as little as 50 ns.

Tests over a 0° to 70°C temperature range indicate that the MPY-16HJ multiplier typically requires 125 ns to calculate 32-bit products.* The worst-case data sheet specification for non-pipelined multiplies is 175 ns.

In light of the above, the nominal 200 ns interval cited earlier for forming reciprocals and for doing multiplies continues to be a reasonable estimate.

Beyond questions of operating speed, several test cases have been run to verify the numerical validity of the basic design, with particular emphasis on the effects of approximation and quantization errors. Appendix G discusses the results of two test cases, including such issues as table memory zone selection, scaling, and error calculations.

The integrated circuit placement diagram of Fig. 8 is indicative of the physical size of the divider. The layout shown requires about 12 in.^{2} of circuit board area.

Total D.C. power dissipation for the divider using available componentry is slightly in excess of 8 watts. This number can be reduced to 5 watts in the future by employing SPROMs which have a fast power-down capability. Raytheon currently offers some of the needed devices which have a built-in power switch, and can potentially save 70% of PROM power.

*The MPY-16HJ has been thoroughly tested at several temperatures using all possible input operand combinations as part of a separate characterization effort.

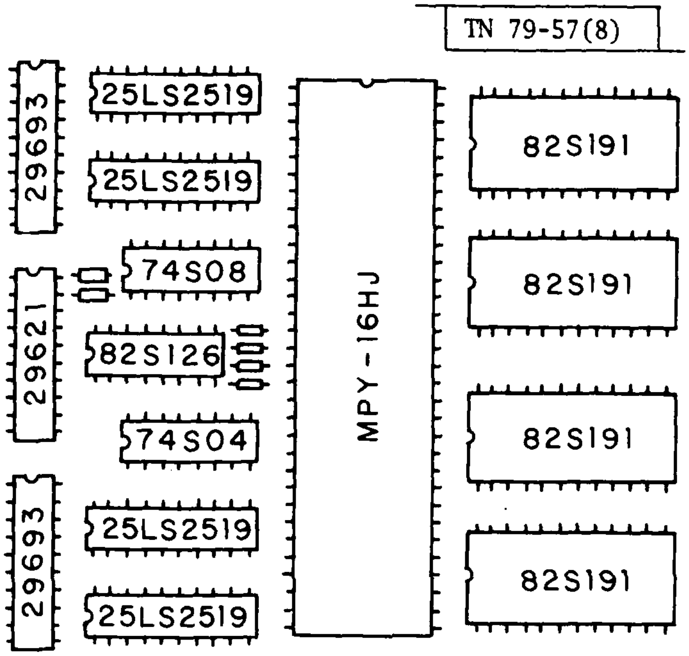


Fig. 8. Integrated Circuit placement diagram for digital divider (actual size).

VII. SUMMARY

For those applications where fast, approximate division is essential, the reciprocal table look-up/multiplication approach to generating quotients is a viable alternative. The design described in this paper is of tractable size (15 DIPs) and power (8 watts), and provides double-precision outputs having total errors no greater than 0.4%. If produced in significant quantities, the device count could be further reduced through the use of semi-custom integrated circuits, particularly for the address mapping function. Better accuracies may be obtained at the expense of additional table storage. For each halving of the approximation error, twice as many PROM table entries are needed.

The hardware implementation shown requires less than 200 ns for reciprocal formation and for each subsequent multiplication. This speed regime is well matched to the capabilities of bipolar (LSTTL) microcomputers or to dedicated hardware systems running with 5 MHz throughput rates.

APPENDIX A

FORTRAN PROGRAMS FOR GENERATING TABLES 3 AND 4

```
      REAL*8 R,RA,RAQ,EA,ET,EQ,ERROR,EPEAK,XPEAK,EAP,ETP,EQP
      REAL*8 DEL,RDEL,X1,X2
      WRITE(6,1)
    1 FORMAT(//,1X,'ERROR ANALYSIS FOR APPROXIMATE 1/X',/)
      WRITE(6,2)
    2 FORMAT(1X,'DATA TABULATED RE MAX INPUT X - SINGLE PRECISION',/)
      WRITE(6,3)
    3 FORMAT(1X,'EA,EQ,ET=APPROX,QUANT,& TOTAL WORST-CASE ERRORS',//)
      WRITE(6,4)
    4 FORMAT(3X,'IMAX',4X,'IPK',6X,'XMAX',9X,'XPK',9X,'EA',8X,'EQ',8X,
     1'ET',//)
      DO 10 L=1,16
      IM=2**(L-1)
      XM=(IM-1.)*2.**(-15)
      EPEAK=-1.
      I1=1
      I2=128
      J=1
      IMAX=2**(L-1)
      IEND=128
      IF(IMAX.LT.IEND)IEND=IMAX
      DO 20 I=1,IEND
      IF(I.GT.2)GO TO 21
      R=1.-(2.**(-15))
      GO TO 22
   21 R=1./(I-1)
   22 RA=R
      CALL QUANT(RA,RAQ)
      GO TO 50
   20 CONTINUE
      IF(I.EQ.IMAX)GO TO 40
   15 J=J+1
      I3=2*(I2-I1)+1
      I1=I2+1
      I2=I1+I3
      IF(IMAX.LT.I2)I2=IMAX
      IF(I2.EQ.32640)I2=32768
      DO 30 I=I1,I2
      R=1./(I-1)
      DEL=2.**(J-1)
      IF(I.GE.32513)DEL=256.
      IDEL=DEL
      RDEL=1./DEL
      X1=I/DEL
      IX1=I/IDEL
```

ERROR16 FORTRAN

```
   X2=X1-IX1
   IF(X2.NE.RDEL)GO TO 30
   RA=2./(2.*(I-1)+DEL-1.)
   CALL QUANT(RA,RAQ)
   GO TO 50
30 CONTINUE
   IF(I.EQ.IMAX)GO TO 40
   GO TO 15
50 EA=(R-RA)/R
   ET=(R-RAQ)/R
   EQ=ET-EA
   ERROR=ET
   IF(ET.LT.0)ERROR=-ET
   IF(ERROR.LE.EPEAK)GO TO 51
   EPEAK=ERROR
   IPEAK=I
   XPEAK=I*2.**(-15)
   EAP=EA
   ETP=ET
   EQP=EQ
51 IF(J.EQ.1)GO TO 20
   GO TO 30
40 WRITE(6,41)IM,IPEAK,XM,XPEAK,EAP,EQP,ETP
41 FORMAT(1X,I6,1X,I6,1X,F12.9,1X,F12.9,1X,F9.6,1X,F9.6,1X,F9.6)
10 CONTINUE
   STOP
   END
   SUBROUTINE QUANT(RA,RAQ)
   REAL*8 RA,RAQ,RS,RR,C
   DIMENSION CO(16)
   RS=RA
   RC=1.
   IF(RS.GE.0)GO TO 60
   RC=-1.
   RS=-RS
60 C=0
   DO 61 N=2,16
   CO(N)=2.**(-(N-1))
   IF(RS.LT.CO(N))GO TO 62
   RS=RS-CO(N)
   C=C+CO(N)
62 RAQ=RC*C
61 CONTINUE
   RETURN
   END
```

ERROR16 FORTRAN

```
      REAL*8 R,RA,RAQ,EA,ET,EQ,ERROR,EPEAK,XPEAK,EAP,ETP,EQP
      REAL*8 DEL,RDEL,X1,X2
      WRITE(6,1)
    1 FORMAT(//,1X,'ERROR ANALYSIS FOR APPROXIMATE 1/X',/)
      WRITE(6,2)
    2 FORMAT(1X,'DATA TABULATED RE MAX INPUT X - DOUBLE PRECISION',/)
      WRITE(6,3)
    3 FORMAT(1X,'EA,EQ,ET=APPROX,QUANT,& TOTAL WORST-CASE ERRORS',//)
      WRITE(6,4)
    4 FORMAT(3X,'IMAX',4X,'IPK',6X,'XMAX',9X,'XPK',9X,'EA',8X,'EQ',8X,
     1'ET',//)
      DO 10 L=1,16
      IM=2**(L-1)
      XM=(IM-1.)*2.**(-15)
      EPEAK=-1.
      I1=1
      I2=128
      J=1
      IMAX=2**(L-1)
      IEND=128
      IF(IMAX.LT.IEND)IEND=IMAX
      DO 20 I=1,IEND
      IF(I.GT.2)GO TO 21
      R=1.-(2.**(-15))
      GO TO 22
   21 R=1./(I-1)
   22 RA=R
      CALL QUANT(RA,RAQ)
      GO TO 50
   20 CONTINUE
      IF(I.EQ.IMAX)GO TO 40
   15 J=J+1
      I3=2*(I2-I1)+1
      I1=I2+1
      I2=I1+I3
      IF(IMAX.LT.I2)I2=IMAX
      IF(I2.EQ.32640)I2=32768
      DO 30 I=I1,I2
      R=1./(I-1)
      DEL=2.**(J-1)
      IF(I.GE.32513)DEL=256.
      IDEL=DEL
      RDEL=1./DEL
      X1=I/DEL
      IX1=I/IDEL
```

ERROR32 FORTRAN

```
      X2=X1-IX1
      IF(X2.NE.RDEL)GO TO 30
      RA=2./(2.*(I-1)+DEL-1.)
      CALL QUANT(RA,RAQ)
      GO TO 50
   30 CONTINUE
      IF(I.EQ.IMAX)GO TO 40
      GO TO 15
   50 EA=(R-RA)/R
      ET=(R-RAQ)/R
      EQ=ET-EA
      ERROR=ET
      IF(ET.LT.0)ERROR=-ET
      IF(ERROR.LE.EPEAK)GO TO 51
      EPEAK=ERROR
      IPEAK=I
      XPEAK=I*2.**(-15)
      EAP=EA
      ETP=ET
      EQP=EQ
   51 IF(J.EQ.1)GO TO 20
      GO TO 30
   40 WRITE(6,41)IM,IPEAK,XM,XPEAK,EAP,EQP,ETP
   41 FORMAT(1X,I6,1X,I6,1X,F12.9,1X,F12.9,1X,F9.6,1X,F9.6,1X,F9.6)
   10 CONTINUE
      STOP
      END
      SUBROUTINE QUANT(RA,RAQ)
      REAL*8 RA,RAQ,RS,RR,C
      DIMENSION CO(32)
      RS=RA
      RC=1.
      IF(RS.GE.0)GO TO 60
      RC=-1.
      RS=-RS
   60 C=0
      DO 61 N=2,32
      CO(N)=2.**(-(N-1))
      IF(RS.LT.CO(N))GO TO 62
      RS=RS-CO(N)
      C=C+CO(N)
   62 RAQ=RC*C
   61 CONTINUE
      RETURN
      END
```

ERROR32 FORTRAN

APPENDIX B
HEXADECIMAL DATA FILES FOR PROM A, PROM B, AND PMUX PROGRAMMING

PMUX DATA

```
7777777677777765
7777765377776537
7776537777653777
7653777765377777
5377777737777777
0000000000000000
0000000000000000
0000000000000000
0000000000000000
0000000000000000
0000000000000000
0000000000000000
0000000000000000
0000000000000000
0000000000000000
0000000000000000
```

PROMA DATA

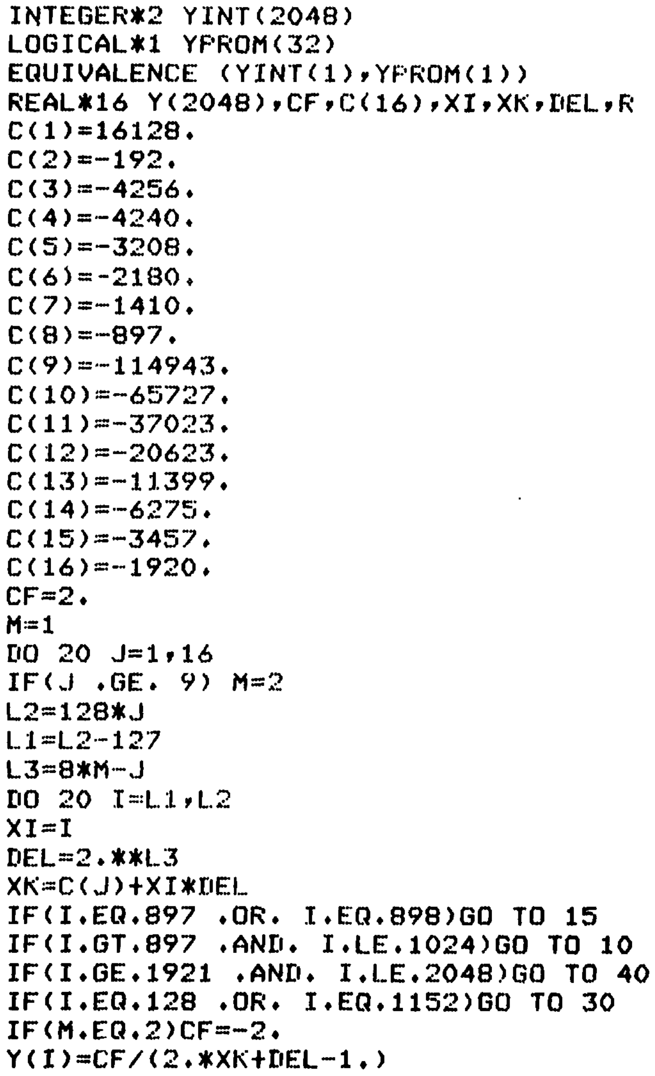
```
7665555444444443
3333333333333332
2222222222222222
2222222222222221
1111111111111111
1111111111111111
1111111111111111
1111111111111110
0000000000000000
0000000000000000
0000000000000000
0000000000000000
0000000000000000
0000000000000000
0000000000000000
0000000000000000
```

PROMB DATA

```
00804040C0C020202020E0E0E0E01010
101050505050B0B0B0B0F0F0F0F00808
08084848484828282828686868689898
9898D8D8D8D8B8B8B8B8F8F8F8F80404
04044444444424242424646464641414
14145454545434343434747474748C8C
8C8CCCCCCCCCACACACACECECECEC9C9C
9C9CDCDCDCDCBCBCBCBCFCFCFCFC0202
02024242424222222222626262621212
12125252525232323232727272720A0A
0A0A4A4A4A4A2A2A2A2A6A6A6A6A1A1A
1A1A5A5A5A5A3A3A3A3A7A7A7A7A8686
8686C6C6C6C6A6A6A6A6E6E6E6E69696
9696D6D6D6D6B6B6B6B6F6F6F6F68E8E
8E8ECECECECEAEAEAEAEEEEEEEEE9E9E
9E9EDEDEDEDEBEBEBEBEFEFEFEFE0000
00004040404020202020606060601010
10105050505030303030707070700808
08084848484828282828686868681818
18185858585838383838787878780404
04044444444424242424646464641414
14145454545434343434747474740C0C
0C0C4C4C4C4C2C2C2C2C6C6C6C6C1C1C
1C1C5C5C5C5C3C3C3C3C7C7C7C7C8282
8282C2C2C2C2A2A2A2A2E2E2E2E29292
9292D2D2D2D2B2B2B2B2F2F2F2F28A8A
8A8ACACACACAAAAAAAAAEAEAEAEA9A9A
9A9ADADADADABABABABAFAFAFAFA8686
8686C6C6C6C6A6A6A6A6E6E6E6E69696
9696D6D6D6D6B6B6B6B6F6F6F6F68E8E
8E8ECECECECEAEAEAEAEEEEEEEEE9E9E
9E9EDEDEDEDEBEBEBEBEFEFEFEFE8181
```

APPENDIX C

FORTRAN PROGRAM FOR CALCULATING SINGLE-PRECISION TABLE ENTRIES



```
INTEGER*2 YINT(2048)
LOGICAL*1 YFROM(32)
EQUIVALENCE (YINT(1),YFROM(1))
REAL*16 Y(2048),CF,C(16),XI,XK,DEL,R
C(1)=16128.
C(2)=-192.
C(3)=-4256.
C(4)=-4240.
C(5)=-3208.
C(6)=-2180.
C(7)=-1410.
C(8)=-897.
C(9)=-114943.
C(10)=-65727.
C(11)=-37023.
C(12)=-20623.
C(13)=-11399.
C(14)=-6275.
C(15)=-3457.
C(16)=-1920.
CF=2.
M=1
DO 20 J=1,16
IF(J .GE. 9) M=2
L2=128*J
L1=L2-127
L3=8*M-J
DO 20 I=L1,L2
XI=I
DEL=2.**L3
XK=C(J)+XI*DEL
IF(I.EQ.897 .OR. I.EQ.898)GO TO 15
IF(I.GT.897 .AND. I.LE.1024)GO TO 10
IF(I.GE.1921 .AND. I.LE.2048)GO TO 40
IF(I.EQ.128 .OR. I.EQ.1152)GO TO 30
IF(M.EQ.2)CF=-2.
Y(I)=CF/(2.*XK+DEL-1.)
```

TABLE1 FORTRAN

```
      GO TO 20
   10 Y(I)=1./XK
   15 Y(897)=1.-(2.**(-15))
      Y(898)=Y(897)
      GO TO 20
   40 Y(I)=-1./XK
      GO TO 20
   30 Y(I)=CF/(2.*XK+255.)
   20 CONTINUE
      DO 70 I=1,2048
      WRITE(3,3)Y(I)
    3 FORMAT(F36.33)
      N=1
      ICUM=0
      R=Y(I)
      IF(Y(I).GE.0)GO TO 25
      N=-1
      R=-Y(I)
   25 R=R-2.**(-15)
      IF(R.LT.0)GO TO 35
      ICUM=ICUM+1
      GO TO 25
   35 YINT(I)=N*ICUM
   70 WRITE(4,4)YINT(I)
    4 FORMAT(I8)
      DO 60 I=1,4065,32
      WRITE(1,2) YPROM(I),YPROM(I+2),YPROM(I+4),YPROM(I+6),
     1YPROM(I+8),YPROM(I+10),YPROM(I+12),YPROM(I+14),
     1YPROM(I+16),YPROM(I+18),YPROM(I+20),YPROM(I+22),
     1YPROM(I+24),YPROM(I+26),YPROM(I+28),YPROM(I+30)
   60 WRITE(2,2) YPROM(I+1),YPROM(I+3),YPROM(I+5),YPROM(I+7),
     1YPROM(I+9),YPROM(I+11),YPROM(I+13),YPROM(I+15),
     1YPROM(I+17),YPROM(I+19),YPROM(I+21),YPROM(I+23),
     1YPROM(I+25),YPROM(I+27),YPROM(I+29),YPROM(I+31)
    2 FORMAT(32Z2)
      STOP
      END
```

TABLE1 FORTRAN

APPENDIX D
HEXADECIMAL DATA FILES FOR PROM 3 AND PROM 4 PROGRAMMING

```
02010101010101010101010101010101
01010101010101010101010101010101
01010101010101010101010101010101
01010101010101010101010101010101
01010101010101010101010101010101        8+
01010101010101010101010101010101
01010101010101010101010101010101
01010101010101010101010101010101
04040303030303030303030303030303
03030303030303030303030303030303
03030303030303030303030303020202
02020202020202020202020202020202
02020202020202020202020202020202        7+
02020202020202020202020202020202
02020202020202020202020202020202
02020202020202020202020202020202
08080808070707070707070707070707
07070707070706060606060606060606
06060606060606060606060606060605
05050505050505050505050505050505
05050505050505050505050505050505        6+
05040404040404040404040404040404
04040404040404040404040404040404
04040404040404040404040404040404
11101010101010100F0F0F0F0F0F0F0F
0F0E0E0E0E0E0E0E0E0E0D0D0D0D0D0D
0D0D0D0D0D0D0C0C0C0C0C0C0C0C0C0C
0C0C0C0B0B0B0B0B0B0B0B0B0B0B0B0B        5+
0B0B0A0A0A0A0A0A0A0A0A0A0A0A0A0A
0A0A0A0A0A0909090909090909090909
09090909090909090909090908080808
08080808080808080808080808080808
```

BYTE3 HEX

```
24242323232222222222212121202020
1F1F1F1F1E1E1E1E1E1D1D1D1D1C1C1C
1C1C1B1B1B1B1B1B1A1A1A1A1A1A1919
19191919181818181818171717171717          4+
17171616161616161616161515151515
15151514141414141414141414131313
13131313131313131212121212121212
12121212111111111111111111111111
555453525150504F4E4D4D4C4B4A4A49
48484747464545444443424241414040
3F3F3E3E3D3D3C3C3C3B3B3A3A393939
38383737373636363535353434343333          3+
33323232313131303030302F2F2F2E2E
2E2E2D2D2D2D2C2C2C2C2B2B2B2B2B2A
2A2A2A29292929292828282828272727
27272626262626262525252525252424
FFFBF7F3F0ECE9E5E2DFDCD9D6D4D1CE
CCC9C7C4C2C0BDBBB9B7B5B3B1AFADAC
AAA8A6A5A3A1A09E9D9B9A9897959493
91908F8E8C8B8A898887868483828180
7F7E7D7C7B7A7A797877767574737372          2+
7170706F6E6D6D6C6B6A6A6968686766
66656464636362616160605F5F5E5E5D
5C5C5B5B5A5A59595858575757565655
FFFF00AA0099554900038CCA2AAD82488
00871CBC6618D190551EECBD92694421
00E0C3A88E755E48331F0CFAE8D8C8B9
AA9C8F82766A5E53493E342B22191008          1+
00F8F0E9E1DAD4CDC7C0BAB4AFA9A49E
99948F8A86817D7874706C6864605C58
55514E4A4744413E3B3835322F2C2927
24211F1C1A181513110E0C0A08060402
```

BYTE3 HEX

```
FEFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF      8-
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FCFCFDFDFDFDFDFDFDFDFDFDFDFDFDFD
FDFDFDFDFDFDFDFDFDFDFDFDFDFDFDFD
FDFDFDFDFDFDFDFDFDFDFDFDFDFEFEFE
FEFEFEFEFEFEFEFEFEFEFEFEFEFEFEFE      7-
FEFEFEFEFEFEFEFEFEFEFEFEFEFEFEFE
FEFEFEFEFEFEFEFEFEFEFEFEFEFEFEFE
FEFEFEFEFEFEFEFEFEFEFEFEFEFEFEFE
FEFEFEFEFEFEFEFEFEFEFEFEFEFEFEFE
F8F8F8F8F9F9F9F9F9F9F9F9F9F9F9F9
F9F9F9F9F9F9FAFAFAFAFAFAFAFAFAFA
FAFAFAFAFAFAFAFAFAFAFAFAFAFAFAFB
FBFBFBFBFBFBFBFBFBFBFBFBFBFBFBFB      6-
FBFBFBFBFBFBFBFBFBFBFBFBFBFBFBFB
FBFCFCFCFCFCFCFCFCFCFCFCFCFCFCFC
FCFCFCFCFCFCFCFCFCFCFCFCFCFCFCFC
FCFCFCFCFCFCFCFCFCFCFCFCFCFCFCFC
F0F0F0F0F0F0F0F0F1F1F1F1F1F1F1F1
F1F2F2F2F2F2F2F2F2F2F3F3F3F3F3F3
F3F3F3F3F3F3F4F4F4F4F4F4F4F4F4F4
F4F4F4F5F5F5F5F5F5F5F5F5F5F5F5F5
F5F5F6F6F6F6F6F6F6F6F6F6F6F6F6F6      5-
F6F6F6F6F6F7F7F7F7F7F7F7F7F7F7F7
F7F7F7F7F7F7F7F7F7F7F7F7F8F8F8F8
F8F8F8F8F8F8F8F8F8F8F8F8F8F8F8F8
```

BYTE3 HEX

```
DCDCDDDDDDDEDEDEDFDFDFDFE0E0E0E0
E1E1E1E1E2E2E2E2E3E3E3E3E3E4E4E4
E4E4E5E5E5E5E5E5E6E6E6E6E6E7E7E7
E7E7E7E7E8E8E8E8E8E8E8E9E9E9E9E9          4-
E9E9EAEAEAEAEAEAEAEAEBEBEBEBEBEB
EBEBEBECECECECECECECECECECEDEDED
EDEDEDEDEDEDEDEDEEEEEEEEEEEEEEEE
EEEEEEEFEFEFEFEFEFEFEFEFEFEFEFEF
ACADADAEAFB0B1B1B2B3B4B4B5B6B6B7
B8B8B9BABABBBBBCBDBDBEBEBFBFC0C0
C1C1C2C2C3C3C4C4C5C5C5C6C6C7C7C7
C8C8C9C9C9CACACACBCBCCCCCCCDCDCD
CDCECECECFCFCFD0D0D0D0D1D1D1D2D2          3-
D2D2D3D3D3D3D4D4D4D4D5D5D5D5D6D6
D6D6D6D7D7D7D7D7D8D8D8D8D8D9D9D9
D9D9DADADADADADBDBDBDBDBDBDCDCDC
03070B0F1216191C1F2225282B2E3033
36383B3D3F414446484A4C4E50525455
57595B5C5E5F6163646667686A6B6D6E
6F70727374757778797A7B7C7D7E7F80
8182838485868788898A8A8B8C8D8E8F          2-
8F90919292939495959697979899999A
9B9B9C9C9D9E9E9F9FA0A1A1A2A2A3A3
A4A4A5A5A6A6A7A7A8A8A9A9AAAAABAB
0000560067ABB700C8345E5628DC7800
79E4449AE82F70ABE214436E97BCDF00
203D58728BA2B8CDE1F4061828384756
64717E8A96A2ADB7C2CCD5DEE7F0F800
0810171F262C333940464C51575C6267          1-
6C71767A7F83888C9094989CA0A4A8AB
AFB2B6B9BCBFC2C5C8CBCED1D4D7D9DC
DFE1E4E6E8EBEDEFF2F4F6F8FAFCFE00
```

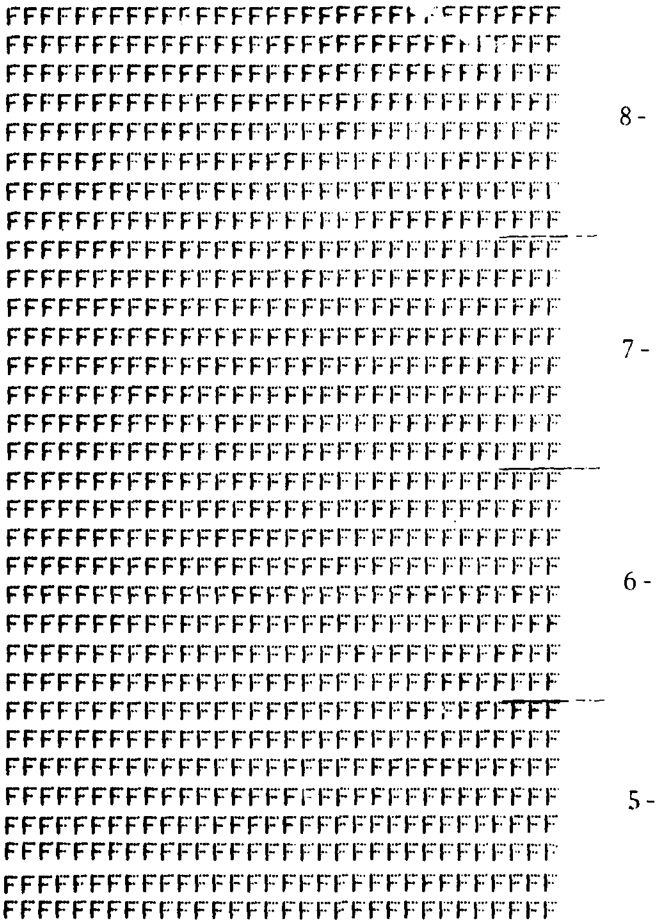
BYTE3 HEX

```
00000000000000000000000000000000
00000000000000000000000000000000
00000000000000000000000000000000
00000000000000000000000000000000    8+
00000000000000000000000000000000
00000000000000000000000000000000
00000000000000000000000000000000
00000000000000000000000000000000
                             ------
00000000000000000000000000000000
00000000000000000000000000000000
00000000000000000000000000000000
00000000000000000000000000000000    7+
00000000000000000000000000000000
00000000000000000000000000000000
00000000000000000000000000000000
00000000000000000000000000000000
                             ------
00000000000000000000000000000000
00000000000000000000000000000000
00000000000000000000000000000000
00000000000000000000000000000000    6+
00000000000000000000000000000000
00000000000000000000000000000000
00000000000000000000000000000000
00000000000000000000000000000000
                             ------
00000000000000000000000000000000
00000000000000000000000000000000
00000000000000000000000000000000
00000000000000000000000000000000    5+
00000000000000000000000000000000
00000000000000000000000000000000
00000000000000000000000000000000
00000000000000000000000000000000
```

BYTE4 HEX

```
00000000000000000000000000000000
00000000000000000000000000000000
00000000000000000000000000000000
00000000000000000000000000000000      4+
00000000000000000000000000000000
00000000000000000000000000000000
00000000000000000000000000000000
00000000000000000000000000000000 ---
00000000000000000000000000000000
00000000000000000000000000000000
00000000000000000000000000000000
00000000000000000000000000000000      3+
00000000000000000000000000000000
00000000000000000000000000000000
00000000000000000000000000000000
00000000000000000000000000000000 ---
00000000000000000000000000000000
00000000000000000000000000000000
00000000000000000000000000000000
00000000000000000000000000000000      2+
00000000000000000000000000000000
00000000000000000000000000000000
00000000000000000000000000000000
00000000000000000000000000000000 ---
7F7F402A20191512100E0C0B0A090908
08070706060605050505040404040404
04030303030303030303030202020202
02020202020202020202020202020202      1+
02010101010101010101010101010101
01010101010101010101010101010101
01010101010101010101010101010101
01010101010101010101010101010101
```

BYTE4 HEX



BYTE4 HEX

```
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF      4 -
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF  - -
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF      3 -
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF  --
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF      2 -
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF  —
80C0D5E0E6EAEDF0F1F3F4F5F6F6F7F8
F8F8F9F9F9FAFAFAFAFBFBFBFBFBFBFC
FCFCFCFCFCFCFCFCFCFDFDFDFDFDFDFD
FDFDFDFDFDFDFDFDFDFDFDFDFDFDFDFE
FEFEFEFEFEFEFEFEFEFEFEFEFEFEFEFE      1 -
FEFEFEFEFEFEFEFEFEFEFEFEFEFEFEFE
FEFEFEFEFEFEFEFEFEFEFEFEFEFEFEFE
FEFEFEFEFEFEFEFEFEFEFEFEFEFEFEFF
```

BYTE4 HEX

APPENDIX E

FORTRAN PROGRAM FOR CALCULATING EXTENDED-PRECISION TABLE ENTRIES

```
      INTEGER*2 YINT(2048)
      LOGICAL*1 YPROM(32)
      EQUIVALENCE (YINT(1),YPROM(1))
      REAL*16 Y(2048),CF,C(16),XI,XK,DEL,R
      C(1)=16128.
      C(2)=-192.
      C(3)=-4256.
      C(4)=-4240.
      C(5)=-3208.
      C(6)=-2180.
      C(7)=-1410.
      C(8)=-897.
      C(9)=-114943.
      C(10)=-65727.
      C(11)=-37023.
      C(12)=-20623.
      C(13)=-11399.
      C(14)=-6275.
      C(15)=-3457.
      C(16)=-1920.
      CF=2.
      M=1
      DO 20 J=1,16
      IF(J .GE. 9) M=2
      L2=128*J
      L1=L2-127
      L3=8*M-J
      DO 20 I=L1,L2
      XI=I
      DEL=2.**L3
      XK=C(J)+XI*DEL
      IF(I.EQ.897 .OR. I.EQ.898)GO TO 15
      IF(I.GT.897 .AND. I.LE.1024)GO TO 10
      IF(I.GE.1921 .AND. I.LE.2048)GO TO 40
      IF(I.EQ.128 .OR. I.EQ.1152)GO TO 30
      IF(M.EQ.2)CF=-2.
      Y(I)=CF/(2.*XK+DEL-1.)
      GO TO 20
   10 Y(I)=1./XK
   15 Y(897)=1.-(2.**(-15))
      Y(898)=Y(897)
      GO TO 20
```

TABLE2 FORTRAN

```
40 Y(I)=-1./XK
   GO TO 20
30 Y(I)=CF/(2.*XK+255.)
20 CONTINUE
   DO 70 I=1,2048
   R=Y(I)
   N=1
   IF(R.GE.0)GO TO 45
   N=-1
   R=-Y(I)
45 R=R-2.**(-15)
   IF(R.LT.0)GO TO 50
   GO TO 45
50 Y(I)=R+2.**(-15)
   IF(N.EQ.-1)Y(I)=-Y(I)
   WRITE(3,3)Y(I)
 3 FORMAT(F36.33)
   N=1
   ICUM=0
   R=Y(I)
   IF(Y(I).GE.0)GO TO 25
   N=-1
   R=-Y(I)
25 R=R-2.**(-30)
   IF(R.LT.0)GO TO 35
   ICUM=ICUM+1
   GO TO 25
35 YINT(I)=N*ICUM
70 WRITE(4,4)YINT(I)
 4 FORMAT(I8)
   DO 60 I=1,4065,32
   WRITE(1,2) YPROM(I),YPROM(I+2),YPROM(I+4),YPROM(I+6),
  1YPROM(I+8),YPROM(I+10),YPROM(I+12),YPROM(I+14),
  1YPROM(I+16),YPROM(I+18),YPROM(I+20),YPROM(I+22),
  1YPROM(I+24),YPROM(I+26),YPROM(I+28),YPROM(I+30)
60 WRITE(2,2) YPROM(I+1),YPROM(I+3),YPROM(I+5),YPROM(I+7),
  1YPROM(I+9),YPROM(I+11),YPROM(I+13),YPROM(I+15),
  1YPROM(I+17),YPROM(I+19),YPROM(I+21),YPROM(I+23),
  1YPROM(I+25),YPROM(I+27),YPROM(I+29),YPROM(I+31)
 2 FORMAT(32Z2)
   STOP
   END
```

TABLE2 FORTRAN

APPENDIX F

HEXADECIMAL DATA FILES FOR PROM 1 AND PROM 2 PROGRAMMING

```
03020A1A315075A2D6105199E73B95F5
5AC637AD29AA30BB4BE07A18BB620EBE
722AE7A76B33FFCFA279533112F6DEC9
B7A89C948E8B8B8D939BA6B4C4D6EB03        8+
1D3A58799DC2EA14406F9FD2063C75AF
EB2969ABEF347BC40E5BA9F8499CF046
9DF650AC0968C8298CF056BC258EF964
D240AF2092057AEF65DD56CF4AC64381
1A0A09193866A3EF48B025A735D1792D
EDB88F715E5557647A9AC4F73379C81F
7FE758D051D96901A047F4A96426EFBF
9470533B2A1E18181D28394F6A8AAFD9        7+
083D75B3F53C87D72B83E040A50E7BEC
60D955D558DF6AF88A1FB753F29439E2
8D3CEEA25A14D191541AE2AD7B4B1DF3
CBA5826142260CF5DFCCBCADA1968E88
EB87452423428 0DD56EDA06E585B79B0
0067E77E2BEFC8B7BBD3004093F972FD
9A4908D9BAABACBDDD0C4B97F25BD256
E88632EAAE7F5C45393843597AA5DB1B        6+
66BA1981F26DF27F16B55D0DC6885123
FCDEC7B8B0B0B7C5DAF71A4475ACEA2E
79C9217EE14AB92EA828AE39C95FFA9B
40EB9A4F08C689511DEEC49E7C5F4631
081E7F291A51CB8987C43FF6E9157A16
E8EF29963504022F8910C29FA6D62EAE
5420112760BC3BDB9D7F81A2E240BB54
0ADBC8D0F33086F67F20D9AA9291A6D2        5+
1369D555EA924F1F02F7001A4785D535
A72ABC5F12D5A788797886A2CC054B9F
006EEA7207A95711D7A98770656570 86
A7D2084993E847B0239F25B44CEE994D
```

BYTE1 HEX

```
EDD26FBFBD66B4A43259166543AC9E14
0C8275E0C217DD11B2BC2E053FDAD42A
DCE748000B6816135DF2D2FB6B221D5C
DD9FA1E15F190F3EA749212F73EB9775      4+
85C637D7A6A2CB20A14D23224A9A12B0
755F6FA3FB7614D5B7BBE0258A0EB173
53506BA2F666F2985A362C3C65A80376
02A560321A1A2F5A9BF25DDD721CD9AA
2A6EF7B59589806B3CE65A8C71FB1FD2
0BBDE06A518E16E1E9248C18C3845632
11EFC48C41DC5BB6EAF2C86AD1FBE385
DDE7A00411C112028CAE65AD85E9D74C      3+
46C2BE372B987CD49FDA849A1C05560C
26A27DB84F428F34318329226D08F32B
AF809AFDA99BD350111459DFA5AAEC6C
281F51BC613D519B1ACFB8D524A6593D
7F421F6A80C8B2B6530F7821A199AA7C
BC1949030306CE20C380247C5A90F358
978B0EFE389C0B66906FE6DD3AE6C9CD
DEE6D08B0428E62EEE199E6E7CBA1A90      2+
0F8CF94D7B793EBEF0CB4556F418B8CD
5039801E0C45C0776583CC39C56B25ED
C0986F420CC872067FDA12240CC64EA1
BC9A3A97AF7E013619A8E0BE3F622380
0000005500CCAA4900C7665D55C42444
00C3E3D733C32E0BAA286297922C2221
001FE14171916B41991861F417C18582
550A1441B1CF4B12494796E511141041
00C00F8DF003A0B43838C80D357B2074      1+
CC870CC830C0FA648B026040420B4191
AA3F05B50AC3A065D8C0E71A25DA0985
24BC2335CBC0F24088AB8A08086E2004
```

BYTE1 HEX

```
0201FAEAD3B48E612EF3B26B1DC96F0F
A93DCC56DA59D348B82389EB48A1F545
91D81C5C97CF0334608AAFD2F00C2439
4B5A666F747777756F675C4E3E2C16FF        8 -
E5C8A988653F17EDC1936230FCC58D52
16D89856I3CD863DF3A75909B86611BC
640BB155F89939D87511AC45DD73099D
30C152E16FFC88129C24AB32B73BBE81_____
F60706F7D7A96B20C65FEA67D83C94E0
20547D9BAEB7B5A8927248I4D89243EC
8C23B33AB930A00869C31560A5E31A4A
7498B6CDDFEAF0F0EADFCFB99E7E582E        7 -
FFCB925412CB8030DC8327C661F88B1B
A62DB131AE279C0E7CE74EB31471CC24
78C91863ABF13474B1EB23588ABAE712
3A6083A4C2DEF81025384957646E767C_____
59BCFD1D1CFCBD60E54E9ACBE1DCBE86
35CD4DB5074368797 55C2FEE9A33BA2E
91E221506F7D7C6A4A1ADB8E33C952CE
3C9DF13873A2C5DCE7E8DCC6A5794302        6 -
B863049C2AAF2A9C0666BE0D5492C8F7
1D3B52616868615 23D20FDD3A26A2CE8
9D4CF59734CB5CE76CEC66DB4AB41978
D32878C40A4C89C1F4234E7495B2CBE0_____
19FE99EAF5BA3B797836B7FC06D66ECE
F9EFB141A0CECC9D40B6012218E68B09
61929F874BED6CCA06221EFBB959DC41
8AB7C8BE9A5C03920865AAD8EFEED8AB        5 -
6811A4228CE2255470796F5426E79634
C13EAA06528EBBD9E8E8D9BC915710BC
5AEA6EE54FADFE437CA9CBE1ECEBDFC9
A77B4403B862029926A92393FA57ACF8
```

BYTE1 HEX

```
4044912B1759F7F3521848E7F77D7BF4
EC6664EAFA97C380D2BB3D59136C6705
4832C503EE86CFC875D6EDBB42827F38
AEE4DB930D4B4F18A7FF200BC14290AB        4-
954ED8325D5B2DD24B9ABFBB8E39BD19
50614D14B83895D1EAE2B970087FD813
2F2E10D57D0A7BD10C2D3320F4AE50DA
4BA5E71227250DDF9B42D451BA0E4E7B
FD260DC65FE9740EC5A8C323D4E25944
AC9C1F3DFF6F957A259EEC1827210CEE
CFB3A09DAED9239128ECE20F761D0636
B17B9708D3FA806ABA72972A2FA89802
E74B309885FAF9849D46804FB4B04676        3-
44B0BD6BBDB35095831C6053F34447FB
648154DE211DD44674610B75A08B39AA
E0DA99206D83620A7DBBC59C40B2F202
8E27FFBBF94E47672C0B75D48CFC806D
14C4C662D86B57D51F68E4C23336179A2
03C305EC992CC37D75C78DE0D88E188B
FC802B0F3ECBC64049EF43522AD96DF0        2-
71FB99573F5CB95E57AB658C2A47EB1E
E84F5C1480A68C38B1FD22240BDA984A
F59C46F8B480615A6FA50083311 0226A
EDAEAFF480577AEEB5D1461643D1C218
0000AB003456B700399AA3AB3CDCBC00
3D1D29CD3DD2F556D89E696ED4DEDF00
E11FBF8F6F95BF67E89F0CE93F7B7EAB
F6ECBF4F31B5EEB7B96A1BEFECF0BF00        1-
40F17310FD604CC8C838F3CB85E08C34
79F438D040069C75FEA0C0BEF5BF6F56
C1FB4BF63D609B284019E6DB26F77BDC
44DDCB35400EC0785576F8F892E0FC00
```

BYTE1 HEX

```
017F7D7B7977757371706E6C6A696765
6462615F5E5C5B59585655545251504E
4D4C4A4948474544434241403F3D3C3B
3A393837363534333231302F2E2D2C2C
2B2A2928272625252423222121201F1E     8+
1D1D1C1B1A1A19181817161515141313
12111110100F0E0E0D0C0C0B0B0A0909
08080707060605040403030202010100
06027E7A76726E6A6763605C5955524F
4B4845423F3C393633302D2A28252220
1D1A181513100E0C09070402007E7B79
777573716F6D6B69676563615F5D5B59
58565452504F4D4B4A4846454342403E     7+
3D3B3A383735343231302E2D2B2A2927
2625232221201E1D1C1B191817161513
1211100F0E0D0C0A0908070605040302
1C140C047C746C645D554E474039322B
251E17110B047E78726C67615B55504A
45403B35302B26211C18130E0905007C
77736F6A66625E5A56524E4A46423E3B
3733302C2825211E1B1714110D0A0704     6+
007D7A7774716E6B686563605D5A5755
524F4D4A474542403D3B383633312E2C
2A272523211E1C1A181513110F0D0B09
006E5C4B3A291808786859493A2C1D0F
007265574A3D3023160A7D7165594E42
372C21160B00766B61574D433930261D
140A01786F675E554D453C342C241C14
0D057D766E676059524A443D362F2822     5+
1B150E08027B756F69635D57514C4640
3B352F2A251F1A150F0A05007B76716C
67625E59544F4B46423D3934302B2723
```

BYTE2 HEX

```
360D653D16704A25015D3A1775533212
7252331476593B1F02664B30157A6047
2D147C644C341D066F58422C17026D58
432F1B0774614E3B28160472604E3D2C      4+
1B0A7A695949392A1A0B7C6D5E4F4132
2416087A6C5F5244372A1D1104786B5F
53473B2F23180C01766B60554A3F352A
20150B01776D63594F453C322920160D
001022374E67021F3E5E01254B721C46
73204F0032651A4F063F78336E2B6928
68286A2D71357B41085019632D784411
5E2C7B4B1B6B3D0F6134085C31065C33      3+
0A6139126B441E78532E0A6643207D5B
391776553515755637187A5C3E210367
4A2E12765B40250B71573D230A715840
2810786049321B046E57412B16006B56
000C2750074B1C796256545D6F0B305E
145319673C187A645349454640596A01
1C3C61093768 1E57145519612D7B4D22
7A553314785E46321F1002776E676360      2+
6061646A717A05111F2F4154687F162F
4A6603224263052 94E741B446D184470
1E4D7D2E6012467B30661E560F48033E
7A3775337232733476387B3F04490F55
00000055004C2A12007166745544E4944
004338503330 3A592A5C275024772204
007C611D1C4F281A191C18055D162C18
555E2E411321686412707B3111174210      1+
000F3E0970730E420E7067741447 0D64
4C454E650C4002522E170B0B162C4C76
2A682F7E5737201109091 01F344F721A
497D38783D08582E08674B3421120802
```

BYTE2 HEX

```
FF81828486888A8C8E8F919395976989A
9B9D9EA0A1A3A4A6A7A9AAABADAEAFB1
B2B3B5B6B7B8BABBBCBDBEBFC0C2C3C4
C5C6C7C8C9CACBCCCDCECFD0D1D2D3D3      8 -
D4D5D6D7D8D9DADADBDCDDDEDEDFE0E1
E2E2E3E4E5E5E6E7E7E8E9EAEAEBECEC
EDEEEEEFEFF0F1F1F2F3F3F4F4F5F6F6
F7F7F8F8F9F9FAFBFBFCFCFDFDFEFEFF
F9FE8285898D9195989C9FA3A6AAADB0    -
B4B7BABDC0C3C6C9CCCFD2D5D7DADDDF
E2E5E7EAECEFF1F4F6F8FBFDFF818486
888A8C8E909294969899A9C9EA0A2A4A6      7 -
A7A9ABADAFB0B2B4B5B7B9BABCBDBFC1
C2C4C5C7C8CACBCDCECFD1D2D4D5D6D8
D9DADCDDDEDFE1E2E3E4E6E7E8E9EAEC
EDEEEFF0F1F2F3F5F6F7F8F9FAFBFCFD
E3EBF3FC848B939BA2AAB1B8BFC6CDD4
DBE1E8EEF5FB81878D93999EA4AAAFB5
BABFC5CACFD4D9DEE3E8ECF1F6FAFF83
888C9095999DA1A5A9ADB1B5B9BDC1C5      6 -
C8CCD0D3D7DADEE1E5E8EBEFF2F5F8FB
FF8285888B8E9194979A9C9FA2A5A8AA
ADB0B2B5B8BABDBFC2C4C7C9CCCED1D3
D5D8DADCDFE1E3E5E7EAECEEF0F2F4F6
8192A4B5C6D7E8F88898A7B6C6D4E3F1
FF8D9BA9B6C3D0DDEAF6838F9BA6B2BE
C9D4DFEAF5FF8A949FA9B3BCC6D0D9E3
ECF5FE879099A2AAB3BBC3CBD3DBE3EB      5 -
F3FB828A9198A0A7AEB5BCC3CAD0D7DE
E4EBF1F8FE848A90969CA2A8AEB4BABF
C5CAD0D5DBE0E5EBF0F5FAFF84898E93
989DA2A7ABB0B5B9BEC2C7CBCFD4D8DC
```

BYTE2 HEX

```
CEF79FC7EE94B9DE83A7CAEC8EB0D1F1
91B1D0EE8CAAC7E4809CB8D3EE88A2BC
D5EE869FB6CEE5FC93A9BFD5EB8095AA
BED2E6FA8EA1B4C7D9EBFE90A1B3C4D5     4-
E6F78798A8B8C8D7E7F68594A3B2C0CF
DDEBF98794A2AFBCC9D6E3F0FD8995A2
AEBAC6D1DDE9F4FF8B96A1ACB6C1CCD6
E1EBF5808A949EA7B1BBC4CED7E1EAF3
9B8BF8E2CBB196F9D9B895F1CAA2F9CE
A1F3C493E0ADF8C28BD298DEA2E5A7E7
A7E6A4E19DD893CC85BCF3AADF94C8FB
ADDF90C1F09FCEFCA9D682AED983ADD7     3-
FFA8D0F79EC4EA90B5DAFEA2C5E88BAD
CFF091B2D2F292B1D0EF8DABC8E6839F
BCD8F48FABC6E0FB95AFC9E2FB94ADC5
DDF58DA5BCD3EA8197ADC3D9EF8499AF
FBE8C595D78DB6D3E5ECE8DAC3A2F9C7
8DCA80AFD6F791A4B2B9BAB6AD9E8AF1
D4B18BDFB0FDC58ACB88C2F8ABDB88B1
D7FB9CBAD5ED8397A8B6C3CDD5DADEDF     2-
DFDCD8D2CAC0B4A79887F5E1CCB59C83
E7CBAD8EEDCBA884DEB790E7BD91E5B8
89DAAAF8C693DFAAF4BD86CD94DA9FE3
A6E9ABECADEDACEAA8E5A2DE99D38DC7
0000AA00B3D5ED008E998BAAB1B6BB00
BCC7AFCCCFC5A6D5A3D8AFDB88DDFB00
839EE2E3B0D7E5E6E3E7FAA2E9D3E7AA
A1D1BEECDE979BED8F84CEEEE8BDEF00     1-
F0C1F68F8CF1BDF18F988BEBB8F29BB3
BAB19AF3BFFDADD1E8F4F4E9D3B389D5
97D081A8C8DFEEF6F6EFE0CBB08DE5B6
82C787C2F7A7D1F798B4CBDEEDF7FD00
```

BYTE2 HEX

APPENDIX G
SAMPLE CALCULATIONS FOR TWO PSEUDODIVIDE TEST CASES

Test Case 1

$X = 3321$ (integer) = CF9 = 0.101 348 876 953 125

$Y = 1672$ (integer) = 688 = 0.051 025 390 625

The exact decimal value for 1672/3321 = Q = 0.503 462 812 405 901 ...

$R = 2^{-15}/X = 0.000\ 301\ 108\ \ldots$

$\hat{Q} = \hat{R}\cdot Y$ is to be calculated in two parts, with $\hat{R} = \hat{R}_m + \hat{R}_1$, and $\hat{Q} = \hat{Q}_m + \hat{Q}_1 = \hat{R}_m\cdot Y + \hat{R}_1\cdot Y$.

Some scaling maneuvers are needed. If we interpret the $\hat{R}_m$ table as being scaled by 2^{-15}, but the $\hat{R}_1$ table as not scaled, then $\hat{Q}_m$ can be retrieved from the multiplier LSP with assigned weights of 2^{-1} to 2^{-15}, and $\hat{Q}_1$ may be found subsequently at the multiplier MSP with weights interpreted as 2^{-16} to 2^{-30}.

Integer X = 3321 is the 88th location in reciprocal zone 5^+, where quantization factor (Δ) is 16 and integer X ranges from 1920 to 3967. From the hex tables, $\hat{R}_m$ = 0009 and $\hat{R}_1$ = 6F88.

Accordingly,

$\hat{Q}_m$ = (0009)(688) = 3AC8 = 0.459 228 515 625

$\hat{Q}_1$ = (6F88)(688) = 2D87040 = 05B0 = 0.044 433 593 750 and

$$\hat{Q} = \hat{Q}_m + \hat{Q}_1 = 0.503\ 622\ 109\ 375$$

$$\begin{aligned}\text{Error } E_T &= (\hat{Q} - Q)/Q \\ &= (0.503\ 662\ 109\ 375 - 0.503\ 462\ 812\ 405)/ \\ &\quad 0.503\ 462\ 812\ 405 \\ &= 0.00039\end{aligned}$$

Test Case 2

$X = -3321$ (integer) = F307 = $-0.101\ 348\ 876\ 953\ 125$

$Y = 1672$ (integer) = 688 = $0.051\ 025\ 390\ 625$

The exact decimal value for $1672/(-3321) = Q =$

$-0.503\ 462\ 812\ 405\ 901 \ldots$

$R = 2^{-15}/X = -0.000\ 301\ 108 \ldots$

Integer $X = -3321$ points to reciprocal zone 5^{-}, 88th location. From the hex tables, $\hat{R}_m$ = FFF7 and $\hat{R}_1$ = 90D9.

Thus

$\hat{Q}_m$ = (FFF7)(688) = 687C538 ⇒ C538 (selecting LSP)

$= -0.459\ 228\ 515\ 625$

$\hat{Q}_1$ = (90D9)(688) = D2A0948 ⇒ FA54 (selecting MSP)

$= -0.044\ 311\ 523\ 437\ 500$

and

$$\hat{Q} = \hat{Q}_m + \hat{Q}_1 = -0.503\ 540\ 039\ 062\ 5$$

$$\begin{aligned}\text{Error } E_T &= (\hat{Q} - Q)/Q \\ &= (-0.5035400390625 + 0.5034628124059)/ \\ &\quad (-0.5034628124059) \\ &= 0.00015\end{aligned}$$

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The design and implementation of hardware for approximate digital division is described. Using standard, commercially-available T^{2}L components, a compact divider has been developed which is well-suited for use within 5 MHz systems having three-state busses. After forming double-length denominator reciprocals by table look-up to high accuracies, single and extended-precision quotients are made available by consecutive numerator multiplications using a single-chip array multiplier.